Let the GAMES Continue

In what would be his centennial year, Martin Gardner, the longtime author of *Scientific American*'s celebrated Mathematical Games column, still inspires mathematicians and puzzle lovers

By Colm Mulcahy and Dana Richards

LIKE A GOOD MAGIC TRICK, A CLEVER PUZZLE CAN INSPIRE AWE, REVEAL MATHEMATICAL TRUTHS and prompt important questions. At least that is what Martin Gardner thought. His name is synonymous with the legendary Mathematical Games column he wrote for a quarter of a century in *Scientific American*. Thanks to his own mathemagical skills, Gardner, who would have celebrated his 100th birthday in October, presented noteworthy mathematics every month with all the wonder of legerdemain and, in so doing, captivated a huge readership worldwide. Many people—obscure, famous and in between—have cited Mathematical Games as informing their decisions to pursue mathematics or a related field professionally.

IN BRIEF

Martin Gardner, who would have turned 100 this month, penned a quarter of a century's worth of Mathematical Games columns in *Scientific American*.

Diverse interests and friends and a formidable intellect helped Gardner to introduce a broad audience to many important topics, including RSA cryptography, the Game of Life, fractals and Penrose tilings.

Many of his columns inspired generations of professional and amateur mathematicians and led to entire communities dedicated to further developments.

His fans continue to meet and generate new results. Old friends and devotees of all ages convene at biennial, invitation-only Gathering 4 Gardner events. Many other people host or attend Celebration of Mind parties worldwide every October in his honor.

Illustration by Tim O'Brien

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Dana Richards is a professor of computer science at George Mason University and is author of an upcoming biography of Martin Gardner. Both knew Gardner and serve on the Centennial Committee for the Gathering 4 Gardner Foundation.

Gardner was a modest man. He never sought out awards and did not aspire to fame. Even so, his written legacy of 100-odd books—reflecting an impressive breadth of knowledge that bridged the sciences and humanities—attracted the attention and respect of many public figures. Pulitzer Prize–winning cognitive scientist Douglas Hofstadter described him as "one of the greatest intellects produced in this country in this century." Paleontologist Stephen Jay Gould remarked that Gardner was "the single brightest beacon defending rationality and good science against the mysticism and anti-intellectualism that surrounds us." And linguist Noam Chomsky described his contribution to contemporary intellectual culture as "unique—in its range, its insight, and its understanding of hard questions that matter."

Although Gardner stopped writing his column regularly in the early 1980s, his remarkable influence persists today. He wrote books and reviews up until his death in 2010, and his community of fans now spans several generations. His readers still host gatherings to celebrate him and mathematical games, and they also produce new results. The best way to appreciate his groundbreaking columns may be simply to reread them—or to discover them for the first time, as the case may be. Perhaps our celebration here of his work and the seeds it planted will spur a new generation to understand just why recreational mathematics still matters in 2014.

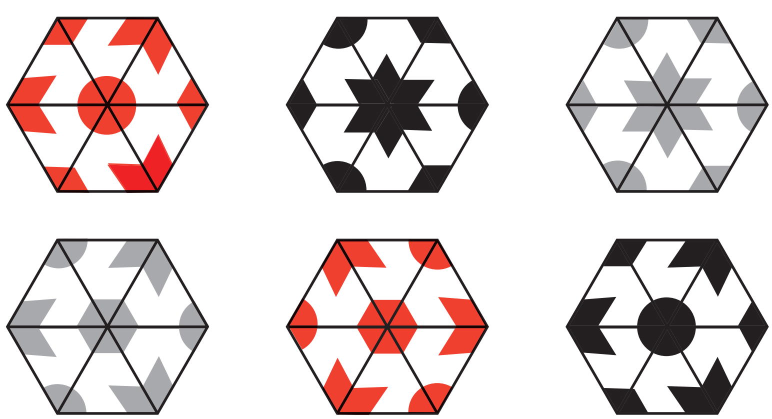
FROM LOGIC TO HEXAFLEXAGONS

FOR ALL HIS FAME in mathematical circles, Gardner was not a mathematician in any traditional sense. At the University of Chicago in the mid-1930s, he majored in philosophy and excelled at logic but otherwise ignored mathematics (although he did audit a course called "Elementary Mathematical Analysis"). He was, however, well versed in mathematical puzzles. His father, a geologist, introduced him to the great turn-of-the-century puzzle innovators Sam Loyd and Henry Ernest Dudeney. From the age of 15, he published articles regularly in magic journals, in which he often explored the overlap between magic and topology, the branch of mathematics that analyzes the properties that remain unchanged when shapes are stretched, twisted or deformed in some other way without tearing. For example, a coffee mug with a handle and a doughnut (or bagel) are topologically the same because both are smooth surfaces with one hole.

In 1948 Gardner moved to New York City, where he became friends with Jekuthiel Ginsburg, a mathematics professor at Yeshiva University and editor of *Scripta Mathematica,* a quarterly journal that sought to extend the reach of mathematics to the general reader. Gardner wrote a series of articles on mathematical magic for the journal and, in due course, seemed to fall under the influence of Ginsburg's argument that "a person does not have to be a painter to enjoy art, and he doesn't have to be a musician to enjoy good music. We want to prove that he doesn't have to be a professional mathematician to enjoy mathematical forms and shapes, and even some abstract ideas."

In 1952 Gardner published his first article in *Scientific American* about machines that could solve basic logic problems. Editor Dennis Flanagan and publisher Gerard Piel, who had taken charge of the magazine several years earlier, were eager to publish more math-related material and became even more interested after their colleague James Newman authored a surprise best seller, *The World of Mathematics,* in 1956. That same year Gardner sent them an article about hexaflexagons—folding paper structures with properties that both magicians and topologists had started to explore. The article was readily accepted, and even before it hit newsstands in December, he had been asked write a monthly column in the same vein.

Gardner's early entries were fairly elementary, but the mathematics became deeper as his understanding—and that of his readers—grew. In a sense, Gardner operated his own sort of social media network but at the speed of the U.S. mail. He shared information among people who would otherwise have worked in isolation, encouraging more research and more findings. Since his university days, he had maintained extensive and meticulously organized files. His network helped him to extend those files and to garner a wide circle of friends, eager to contribute ideas. Virtually anyone who wrote to him got a detailed reply, almost as though they had queried a search engine. Among his correspondents and associates were mathematicians John Hor-



SIX DIFFERENT PICTURES can be made to appear after a single decorated strip of paper is folded into a flat hexagonal structure called a hexahexaflexagon and then twisted and reflattened multiple times, as Gardner demonstrated in *Scientific American* in December 1956. (For a cutout you can use to make your own hexaflexagon, go to ScientificAmerican.com/oct2014/gardner)

ton Conway and Persi Diaconis, artists M. C. Escher and Salvador Dali, magician and skeptic James Randi, and writer Isaac Asimov.

Gardner's diverse alliances reflected his own eclectic interests—among them, literature, conjuring, rationality, physics, science fiction, philosophy and theology. He was a polymath in an age of specialists. In every essay, it seems, he found a connection between his main subject and the humanities. Such references helped many readers to relate to ideas they might have otherwise ignored. For instance, in an essay on "Nothing," Gardner went far beyond the mathematical concepts of zero and the empty set—a set with no members—and explored the concept of nothing in history, literature and philosophy. Other readers flocked to Gardner's column because he was such a skillful storyteller. He rarely prepared an essay on a single result, waiting instead until he had enough material to weave a rich tale of related insights and future paths of inquiry. He would often spend 20 days on research and writing and felt that if he struggled to learn something, he was in a better position than an expert to explain it to the public.

Gardner translated mathematics so well that his columns often prompted readers to pursue topics further. Take housewife Marjorie Rice, who, armed with a high school diploma, used what she learned from a Gardner column to discover several new types of tessellating pentagons, five-sided shapes that fit together like tiles with no gaps. She wrote to Gardner, who shared the result with mathematician Doris Schattschneider to verify it. Gardner's columns seeded scores of new findings—far too many to list. In 1993, though, Gardner himself identified the five columns that generated the most reader response: ones on Solomon W. Golomb's polyominoes, Conway's Game of Life, the nonperiodic tilings of the plane discovered by Roger Penrose of the University of Oxford, RSA cryptography and Newcomb's paradox [*see box on next page*].

POLYOMINOES AND LIFE

PERHAPS SOME OF THESE SUBJECTS proved so popular because they were easy to play with at home, using common items such as chessboards, matchsticks, cards or paper scraps. This was certainly the case when, in May 1957, Gardner described the work by Golomb, who had recently explored the properties of polyominoes, figures made by joining multiple squares side by side; a domino is a polyomino with two squares, a tromino has three, a tetromino has four, and so forth. They turn up in all kinds of tilings, logic problems and popular games, including modern-day video games such as Tetris. Puzzlers were already familiar with these shapes, but as Gardner reported, Golomb took the topic further, proving theorems about what arrangements were possible.

Certain polyominoes also appear as patterns in the Game of Life, invented by Conway and featured in *Scientific American* in October 1970. The game involves "cells," entries in a square array marked as "alive" or "dead," that live (and can thus proliferate) or die according to certain rules—for instance, cells with two or three neighbors survive, whereas those with no, one, or four or more neighbors die. "Games" start off with some initial configuration, and then these groupings evolve according to the rules. Life was part of a fledgling field that used "cellular automata" (rule-driven cells) to simulate complex systems, often in intricate detail. Conway's insight was that a trivial two-state automaton, which he designed by hand, contained the ineffable potential to model complex and evolutionary behavior.

PUZZLE SAMPLER

Test Yourself

Recreational math puzzles fall into many broad categories and solving them draws on a variety of talents, as the examples here, some of which are classics, show. (For the answers, go to ScientificAmerican.com/oct2014/gardner)

Some puzzles call for little more than basic reasoning. For instance, consider this brainteaser: There are three on/off switches on the ground floor of a building. Only one operates a single lightbulb on the third floor. The other two switches are not connected to anything. Put the switches in any on/off order you like. Then go to the third floor to see the bulb. Without leaving the third floor, can you figure out which switch is genuine? You get only one try.

Cryptarithms serve up harder tests of a puzzler's abilities. In these problems, each letter corresponds to a single digit. For instance, can you figure out which digit each letter represents to make the sum at the right work?

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    SEVEN
    SEVEN
    SEVEN
    SEVEN
    SEVEN
    SEVEN
+   SEVEN
---------
  FORTY9
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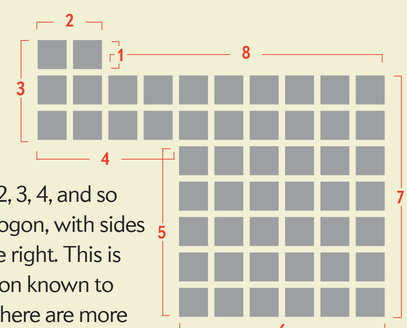
A knack for visualization is helpful for solving geometric stumpers. Can you picture a solid pyramid consisting of a square base and four equilateral triangles, alongside a solid tetrahedron with four faces identical to those of the pyramid's triangles? Now glue one triangle face of the pyramid to a triangle on the tetrahedron. How many faces does the resultant polyhedron have? It's not seven!

Puzzlers, like mathematicians, must sometimes solve challenges that reflect general problems or require the construction of logical proofs. Think about the class of polygons known as serial isogons. All adjacent sides meet at 90 degrees, and the sides are of increasing length: 1, 2, 3, 4, and so on. The simplest isogon, with sides 1–8, is shown at the right. This is the only serial isogon known to tile the plane. But there are more isogons. Can you prove that the number of their sides must always be a multiple of 8?



The properties of chess pieces play a part in many challenges, including in a group of problems about unattacked queens. Imagine three white queens and five black queens on a 5 × 5 chessboard. Can you arrange them so that no queen of one color can attack a queen of the other color? There is only one solution, excluding reflections and rotations.

Newcomb's Paradox: Who Wants to Be a Millionaire?

Martin Gardner read about a problem known as Newcomb's paradox in a 1969 paper by philosopher Robert Nozick and made it the subject of columns in July 1973 and March 1974. This thought experiment, created by theoretical physicist William Newcomb, draws on the mystery of determinacy and free will and is still actively debated in philosophical circles.

Players are pitted against a Predictor—a superintelligent alien, psychic, all-knowing deity—who is gifted at foretelling the player's actions. The player, unaware of the predictions, is presented with two boxes: one that always contains $1,000, call it box A, and another, box B, that might contain $1,000,000. He or she has the choice of taking just box B or taking both boxes. Before the game starts, the Predictor anticipates what the player will do. If the Predictor thinks the player will take only box B, then that box will contain the million-dollar reward. If the Predictor thinks the player will take both boxes, box B will hold nothing.

The paradox arises because two opposing strategies for winning the most money both seem logical. The first strategy argues that taking both boxes always yields more money, regardless of the Predictor's prediction. If the Predictor foretells that the player will take both boxes, then the player who chooses both boxes wins $1,000; selecting just box B yields $0 (*table at right*). If the Predictor anticipates that the player will take only box B, the player who chooses both boxes gets $1,001,000; selecting only box B yields a bit less ($1,000,000).

But another argument says that the greatest winnings will always come from taking only box B. It reasons that the player can ignore the instances in which the player's choice differs from the prediction because those moves require the Predictor to make a mistake, which this deity, by definition, is extremely unlikely to do. The choice then is between taking both boxes for $1,000 or only box B for $1,000,000.

Gardner's readers produced bags of commentary, delineating various outcomes, but there is still no resolution as to whether one strategy is ever better than the other. In his original coverage, Nozick commented, "To almost everyone, it is perfectly clear and obvious what should be done. The difficulty is that these people seem to divide almost evenly on the problem, with large numbers thinking that the opposing half is just being silly."

PREDICTED CHOICE	ACTUAL CHOICE	PAYOUT
Both A and B	Both	$1,000
Both A and B	B only	$0
B only	Both	$1,001,000
B only	B only	$1,000,000

After Gardner's column appeared, the Game of Life quickly attracted a cultlike following. "All over the world mathematicians with computers were writing Life programs," Gardner recalled. His dedicated readership soon produced many surprising findings. Mathematicians had long known that a short list of axioms can lead to profound truths, but the Life community in the early 1970s experienced it firsthand. Some 40 years later Life continues to spark discoveries: a new self-constructing pattern known as Gemini—which copies itself and destroys its parent pattern while innovatively moving in an oblique direction—was reported in May 2010, and the first Life replicator that clones itself and its instructions was built in November 2013.

APERIODICITY AND PUBLIC KEYS

CONWAY ALSO INTRODUCED Gardner to the tilings discovered by Penrose, who is a mathematician and physicist, and they became the basis of another blockbuster column, featuring two tile shapes, called kites and darts for their resemblance to those toys [*see illustration on opposite page*]. Given an endless supply of each, combinations of these tiles can cover an infinite stretch of floor without gaps and display a remarkable property called aperiodicity. Ordinary tile shapes—squares, triangles, hexagons—cover the floor in a pattern that repeats periodically. In other words, there are multiple spots in which you might stand, and the pattern in the tiles underneath your feet would be identical. But when kites and darts, or other combinations of two or more Penrose tiles, are arranged according to certain rules, no such recurring patterns appear. These tilings were so beautiful that in January 1977 they graced *Scientific American*'s cover, based on a sketch by Conway.

The community exploring the properties of Penrose tilings has made a number of advances since, including finding that the patterns display a property called self-similarity, also enjoyed by fractals, structures that repeat at different scales. (Fractals, too, gained widespread popularity in large part because of Gardner's December 1976 column about them.) And Penrose tiles have also led to the discovery of quasicrystals, which have an orderly but aperiodic structure. Nobody was more delighted about the connection than Gardner, who commented, "They are wonderful examples of how a mathematical discovery, made with no inkling of its applications, can turn out to have long been familiar to Mother Nature!"

In August 1977 Gardner anticipated another modern-day development: the use of electronic mail for personal communication "in a few decades." This prediction opened a column that introduced the world to RSA cryptography, a public-key cryptosystem based on trapdoor functions—ones that are easy to compute in one direction but not in the opposite direction. Such systems were not new in the mid-1970s, but computer scientists Ron Rivest, Adi Shamir and Leonard Adleman (after whom RSA is named) introduced a different kind of trapdoor using large prime numbers (those divisible only by one and themselves). The security of RSA encryption stemmed from the apparent difficulty of factoring the product of two sufficiently large primes.

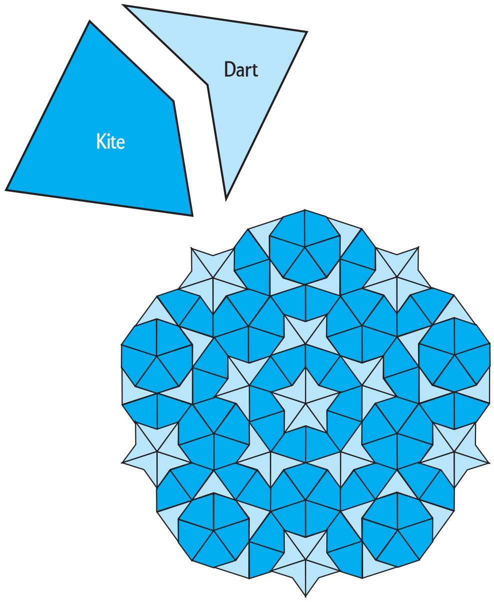
Before publishing their result in an academic journal, Rivest, Shamir and Adleman wrote to Gardner, hoping to reach a large audience quickly. Gardner grasped the significance of their innovation and uncharacteristically rushed a report into print. In the column, he posed a challenge, asking readers to attempt to decode a message that would require them to factor a 129-digit integer, an impossible task at that time. Gardner wisely prefaced the challenge with an Edgar Allan Poe quotation: “Yet it may be roundly asserted that human ingenuity cannot concoct a cipher which human ingenuity cannot resolve.” And indeed, only 17 years later, a large team of collaborators, relying on more than 600 volunteers and 1,600 computers, cracked the code, revealing that the secret message read: “The magic words are squeamish ossifrage.” RSA challenges continued for many years, ending only in 2007.

AFTER GARDNER

GARDNER’S LOVE OF PLAY went hand in hand with his impish sense of fun. A 1975 April Fools’ Day column featured “six sensational discoveries that somehow or another have escaped public attention.” All were plausible—and false. For instance, he claimed that Leonardo da Vinci invented the flush toilet. Allusions to “Ms. Birdbrain” and the psychic-powered “Ripoff rotor” were meant to alert readers to the gag nature of the column, but hundreds failed to get the joke and sent Gardner animated letters.

In 1980 Gardner decided to retire his column to concentrate on other writing projects. *Scientific American* quickly introduced a successor: Douglas Hofstadter. He wrote 25 columns, entitled Metamagical Themas—an anagram of Mathematical Games—many of which discussed artificial intelligence, his own specialty. A. K. Dewdney followed, penning seven years of Computer Recreations. Ian Stewart’s Mathematical Recreations column ran for the next decade. Later Dennis Shasha wrote a long series of Puzzling Adventures, based on computing and algorithmic principles, subtly disguised. “Martin Gardner was an impossible act to follow,” Stewart once commented. “What we did try to do was replicate the spirit of the column: to present significant mathematical ideas in a playful mood.”

For the past two decades the spirit of the column has lived on at invitation-only, biennial Gathering 4 Gardner conferences, where mathematicians, magicians and puzzlers assemble to share what they wish they could still share via Mathematical Games. Gardner himself attended the first two. In recent years participants have ranged from old friends, such as Golomb, Conway, Elwyn Berlekamp, Richard Guy and Ronald Graham, to rising stars, such as computer scientist Erik Demaine and video maven Vi Hart, and some very young blood in the form of talented teenagers Neil Bickford, Julian Hunts and Ethan Brown. Following Gardner’s death in 2010, spin-off Celebration of Mind parties, which anyone can attend (or host), have been held all over the world every October in his honor [see “More to Explore,” at right].



PENROSE TILES are remarkable for producing “aperiodic” patterns: given an infinite supply, they will fill the floor without gaps such that the initial configuration never repeats exactly. Gardner wrote about Penrose tiles called kites and darts in January 1977. To ensure aperiodicity, the tiles must be laid according to certain rules. The starting grouping above is named “the infinite star pattern.”

Although Gardner is gone, there are good reasons to take inspiration from his work and to champion recreational mathematics today. Noodling over puzzles and related activities often leads to important discoveries, as shown, if only briefly, in this article. Almost every essay Gardner wrote gave rise to communities of enthusiasts and specialists. A great number of his columns could now be expanded into books—entire shelves of books even. In addition, thinking about a problem from a mathematical perspective can be enormously valuable for clarity and rigor. Gardner never thought of recreational mathematics as a set of mere puzzles. The puzzles were a gateway to a richer world of mathematical marvels.

In his final, retrospective *Scientific American* article in 1998, Gardner reflected that the “line between entertaining math and serious math is a blurry one.… For 40 years I have done my best to convince educators that recreational math should be incorporated into the standard curriculum. It should be regularly introduced as a way to interest young students in the wonders of mathematics. So far, though, movement in this direction has been glacial.”

Today the Internet hosts scores of math-related apps, tutorials and blogs—including many different Game of Life apps of varying quality—and social media can connect like-minded aficionados faster than Gardner ever could. But maybe that speed has a downside: Web-based experiences are perfect for quick “Interesting!” responses, but it takes careful reflection to reach revelatory “Aha!” moments. We believe that part of the success of Gardner’s column was that he and his audience took the trouble to exchange detailed ideas and craft thoughtful answers. Only time will tell if a new community of puzzlers—in a less patient era—will pick up Gardner’s mantle and propel future generations to fresh insights and discoveries. SA

MORE TO EXPLORE

Gathering 4 Gardner Foundation: **http://gathering4gardner.org**

Martin Gardner home page: **www.martin-gardner.org**

Celebration of Mind: **www.celebrationofmind.org**

A Tribute to Martin Gardner, 1914–2010. In-Depth Reports, ScientificAmerican.com, May 25, 2010. **www.scientificamerican.com/report/martin-gardner-1914-2010**

Flexagon but Not Forgotten: Celebrating Martin Gardner’s Birthday. Evelyn Lamb. Observations blog, ScientificAmerican.com, October 19, 2012. **http://blogs.scientificamerican.com/observations/2012/10/19/flexagon-but-not-forgotten**

FROM OUR ARCHIVES

A Quarter-Century of Recreational Mathematics. Martin Gardner; August 1998.

The Great Explicator. Brian Hayes; October 2013.

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